

**WHAT IS CLAIMED IS:**

1. A device manufacturing method comprising:  
providing a beam radiation;  
projecting said beam of radiation onto a substrate having radiation sensitive material; and

specifying a trajectory to be followed by said substrate relative to said projected beam of radiation, said trajectory being characterized at least as a third order mathematical function containing information indicative of at least one of a position and an orientation as a function of time, said trajectory connecting a first state and a second state representing boundary values for said at least position and orientation and for first and second derivatives of said at least position and orientation,

wherein said specified trajectory provides the shortest connection in time between said first and second state and wherein said first derivative, said second derivative, and a third derivative of said specified trajectory are bound by predetermined values.

2. The method of Claim 1, wherein said boundary values for said second derivatives in the first and second state are zero.

3. The method of Claim 1, wherein said projected beam of radiation is configured with a desired pattern in its cross-section, said desired pattern provided by a patterning device.

4. The method of Claim 1, further comprising:  
calculating separate sub-trajectories for respective dimensions of said at least position and orientation, wherein each of said sub-trajectories is characterized at least as a third order mathematical function having the shortest connection time between said first state and said second state in the respective dimension and wherein said first derivative, said second derivative, and said third derivative of each of said sub-trajectories are bound

by predetermined values specified for that sub-trajectory;

selecting one of said sub-trajectories out of said calculated sub-trajectories based on the sub-trajectory having the largest connection time;

prolonging non-selected sub-trajectories to the connection time of the selected sub-trajectory while meeting the requirements of the respective predetermined values for the respective sub-trajectories, and

generating said specified trajectory by combining the selected sub-trajectory and the prolonged sub-trajectories.

5. The method of Claim 4, further including calculating at least one sub-trajectory between a position  $r_b$ , velocity  $v_b$  and zero acceleration of a first state and a second position  $r_e$ , velocity  $v_e$  and zero acceleration of the second state by:

calculating a time parameter  $t_c$  based on:

$$t_c = \frac{r_e - r_b - \Delta r_a - \Delta r_b}{v_i} \quad (F1)$$

wherein  $\Delta r_a$  and  $\Delta r_b$  are given by:

$$\Delta r_a = \frac{1}{2}(v_b + v_i)t_a \quad (F2)$$

$$\Delta r_b = \frac{1}{2}(v_i + v_e)t_b \quad (F3)$$

wherein  $v_i$  is defined by:

$$v_i = \begin{cases} -V & \text{if } r_e - r_b < 0 \\ +V & \text{if } r_e - r_b > 0 \end{cases} \quad (F4)$$

wherein  $V$  is a predetermined value defining the maximum absolute velocity, and wherein  $t_a$  and  $t_b$  are given by:

$$t_a = \begin{cases} 2\sqrt{\frac{|v_i - v_b|}{J}} & \text{if } |v_i - v_b| \leq A^2 / J \\ \frac{|v_i - v_b|}{A} + \frac{A}{J} & \text{if } |v_i - v_b| \geq A^2 / J \end{cases} \quad (F5)$$

$$t_b = \begin{cases} 2\sqrt{\frac{|v_e - v_i|}{J}} & \text{if } |v_e - v_i| \leq A^2 / J \\ \frac{|v_e - v_i|}{A} + \frac{A}{J} & \text{if } |v_e - v_i| \geq A^2 / J \end{cases} \quad (\text{F6})$$

wherein  $A$  is a predetermined value defining the maximum absolute acceleration of the sub-trajectory and wherein  $J$  is a predetermined value defining the maximum absolute jerk of the sub-trajectory;

wherein, if  $t_c$  is greater than zero, then a sub-trajectory of a first kind comprising three parts is calculated, such that:

in the first part, a maximum velocity is built up in the shortest time  $t_a$  within the limitations of said predetermined values for the sub-trajectory,

in the second part, which has a duration of  $t_c$ , the velocity is constant and equals  $v_b$  plus the velocity built up in the first part, and

in the third part, the velocity is decreased to the velocity  $v_e$  in the shortest time  $t_b$  possible within the limitations of the predetermined values of the sub-trajectory; and

wherein, if  $t_c$  is less than zero, a sub-trajectory of a second kind comprising two parts is calculated, such that:

in the first part, a velocity is built up in the shortest time  $t_a$  possible within the limitations of the predetermined values of the sub-trajectory, and

in the second part, the velocity is decreased to the velocity  $v_e$  in the shortest time  $t_b$  possible within the limitations of the predetermined values of the sub-trajectory.

6. The method of Claim 5, further including performing a detection of a gap crossing move based on:

$$v_b > 0 \quad \wedge \quad v_e > 0 \quad \wedge \quad r_e > r_b \quad (\text{F7})$$

$$v_b < 0 \quad \wedge \quad v_e < 0 \quad \wedge \quad r_e < r_b \quad (\text{F8})$$

$$|r_e - r_b| < \Delta r(v_b) + \Delta r(v_e) \quad (\text{F9})$$

wherein the function  $\Delta r(v)$  is given by:

$$\Delta r(v) = \begin{cases} |v| \sqrt{\frac{|v|}{J}} & \text{if } |v| \leq A^2 / J \\ \frac{1}{2} |v| \left( \frac{|v|}{A} + \frac{A}{J} \right) & \text{if } |v| \geq A^2 / J \end{cases} \quad (\text{F10})$$

wherein a gap-crossing-move is detected if at least one of equation (F7) and equation (F8) is satisfied and equation (F9) is satisfied.

7. The method of Claim 6, wherein if  $t_c < 0$ , or if a gap crossing move is detected, the calculation of said sub-trajectory of the second kind comprises calculating four respective types of acceleration profiles associated with four possible sub-trajectories and said selection of the sub-trajectory out of the four possible sub-trajectories which has the shortest connection time,

wherein the first profile type is associated with the first possible sub-trajectory with a connection time  $T$  which follows from the quadratic expression:

$$\left\{ \frac{1}{2} kA \right\} T^2 + \left\{ (v_e + v_b) - \frac{A^2}{kJ} \right\} T + \left\{ -2(r_e - r_b) - \frac{(v_e - v_b)^2}{2kA} \right\} = 0 \quad (\text{F11})$$

wherein the quadratic expression is solved for  $k=1$  and  $k=-1$ , and wherein the time duration of the first part  $t_a$  and the time duration of the second part  $t_b$  follow from:

$$t_a = \frac{1}{2} T + \frac{v_e - v_b}{2kA} \quad (\text{F12})$$

$$t_b = \frac{1}{2} T - \frac{v_e - v_b}{2kA} \quad (\text{F13})$$

wherein (F11) might yield more than one mathematical solution, wherein the mathematical solutions defining the first profile type satisfy:

$$t_a \geq 2A/J \quad (\text{F14})$$

$$t_b \geq 2A/J \quad (\text{F15})$$

$$|v_i| \leq V, \quad (\text{F16})$$

wherein for the first profile type  $v_i$  is given by:

$$v_i = \frac{1}{2}(v_b + v_e) - \frac{A^2}{kJ} + \frac{1}{2}kAT. \quad (\text{F17})$$

wherein the second profile type is associated with the second possible sub-trajectory with a time duration  $t_a$  of the first part which follows from the quartic expression:

$$\left\{ \frac{kJ^2}{32A} \right\} t_a^4 + \left\{ \frac{kJ}{8} \right\} t_a^3 + \left\{ \frac{v_b J}{4A} + \frac{kA}{8} \right\} t_a^2 + \left\{ v_b \right\} t_a + \left\{ \frac{(v_e + v_b)A}{2J} - \frac{v_e^2 - v_b^2}{2kA} - (r_e - r_b) \right\} = 0 \quad (\text{F18})$$

wherein:

$$k = \begin{cases} -1 & \text{if } v_e > v_b \\ +1 & \text{if } v_e < v_b \end{cases}, \quad (\text{F19})$$

wherein the time duration  $t_b$  of the second part and the velocity  $v_i$  at the transition from the first to the second part follow from:

$$v_i = v_b + \frac{1}{4}kJt_a^2 \quad (\text{F20})$$

$$t_b = \frac{J}{4A}t_a^2 - \frac{v_e - v_b}{kA} + \frac{A}{J} \quad (\text{F21})$$

wherein equation (F18) might yield more than one mathematical solution, wherein the mathematical solutions defining the second profile type satisfy:

$$0 \leq t_a \leq 2A/J \quad (\text{F22})$$

$$t_b \geq 2A/J \quad (\text{F23})$$

$$|v_i| \leq V \quad (\text{F24})$$

and wherein the connection time  $T$  of the second possible sub-trajectory follows from  $T = t_a + t_b$ ;

wherein the third profile type is associated with the third sub-trajectory with a time duration  $t_b$  of the second part which follows from the quartic expression:

$$\left\{ \frac{kJ^2}{32A} \right\} t_b^4 + \left\{ \frac{kJ}{8} \right\} t_b^3 + \left\{ \frac{v_e J}{4A} + \frac{kA}{8} \right\} t_b^2 + \left\{ v_e \right\} t_b + \left\{ \frac{(v_e + v_b)A}{2J} + \frac{v_e^2 - v_b^2}{2kA} - (r_e - r_b) \right\} = 0, \quad (F25)$$

wherein:

$$k = \begin{cases} -1 & \text{if } v_e < v_b \\ +1 & \text{if } v_e > v_b \end{cases}, \quad (F26)$$

wherein the time duration  $t_a$  of the first part and the velocity  $v_i$  at the transition from the first to the second part follow from:

$$v_i = v_e + \frac{1}{4} k J t_b^2 \quad (F27)$$

$$t_a = \frac{J}{4A} t_b^2 + \frac{v_e - v_b}{kA} + \frac{A}{J} \quad (F28)$$

wherein (F25) might yield more than one mathematical solution, wherein the mathematical solutions defining the third profile type satisfy:

$$t_a \geq 2A/J \quad (F29)$$

$$0 \leq t_b \leq 2A/J \quad (F30)$$

$$|v_i| \leq V \quad (F31)$$

and wherein the connection time  $T$  of the third possible sub-trajectory follows from  $T = t_a + t_b$ ;

wherein the fourth profile type is associated with the fourth possible sub-trajectory with a time duration  $T$  which follows from the quartic expression:

$$\left\{ \frac{kJ}{16} \right\} T^4 + \left\{ v_e + v_b \right\} T^2 + \left\{ -2(r_e - r_b) \right\} T + \left\{ -\frac{(v_e - v_b)^2}{kJ} \right\} = 0 \quad (F32)$$

wherein the quartic expression is solved for  $k=1$  and  $k=-1$ , and wherein the time duration  $t_a$  of the first part and the time duration  $t_b$  of the second part and the velocity  $v_i$  at the transition from the first to the second part follow from:

$$t_a = \frac{1}{2} T + 2 \frac{v_e - v_b}{kJT} \quad (F33)$$

$$t_b = \frac{1}{2} T - 2 \frac{v_e - v_b}{kJT} \quad (F34)$$

$$v_i = v_b + \frac{1}{4}kJt_a^2 \quad (F35)$$

wherein (F32) might yield more than one mathematical solution, wherein the mathematical solution defining the fourth profile type satisfies:

$$0 \leq t_a \leq 2A/J \quad (F36)$$

$$0 \leq t_b \leq 2A/J \quad (F37)$$

$$|v_i| \leq V \quad (F38)$$

8. The method of Claim 7, wherein if a gap-crossing-move is detected, then, if  $v_b$  and  $v_i$  have the same sign, a selection from the solutions provided by equations (F11-F17), (F18-F24), (F25-F31) and (F32-F38), respectively, is made such that the selection of the solutions define respectively the shortest connection time  $t_1$  and the longest connection time  $t_2$ , wherein  $t_1$  and  $t_2$  define a first connection time interval comprising possible connection times according to which the concerning sub-trajectory can be prolonged, and wherein, if  $v_b$  and  $v_i$  have opposite signs, a selection of the solutions provided by equations (F11-F17), (F18-F24), (F25-F31) and (F32-F38) is made such that the selection defines a connection time  $t_3$  defining a lower limit of a second connection time interval comprising possible connection times according to which the concerning sub-trajectory can be prolonged.

9. The method of Claim 8, wherein calculating a prolonged sub-trajectory comprises calculating a sub-trajectory with lower predetermined value or values than the corresponding predetermined value or values of the sub-trajectory to be prolonged.

10. The method of Claim 9, wherein if at least one of  $v_b$  and  $v_e$  equal 0, calculating a prolonged sub-trajectory comprises inserting into the sub-trajectory to be prolonged a time interval wherein the velocity is zero.

11. The method of Claim 10, wherein calculating a prolonged sub-trajectory comprises lowering the velocity  $v_i$  to increase  $T$ , wherein the connection time  $T$  for the sub-

trajectory is defined by  $T = t_a + t_b + t_c$ , and wherein  $t_a$ ,  $t_b$  and  $t_c$  are given by:

$$t_a = \begin{cases} 2\sqrt{\frac{|v_i - v_b|}{J}} & \text{if } |v_i - v_b| \leq A^2 / J \\ \frac{|v_i - v_b|}{A} + \frac{A}{J} & \text{if } |v_i - v_b| \geq A^2 / J \end{cases} \quad (\text{F39})$$

$$t_b = \begin{cases} 2\sqrt{\frac{|v_e - v_i|}{J}} & \text{if } |v_e - v_i| \leq A^2 / J \\ \frac{|v_e - v_i|}{A} + \frac{A}{J} & \text{if } |v_e - v_i| \geq A^2 / J \end{cases} \quad (\text{F40})$$

$$t_c = \frac{r_e - r_b - \Delta r_a - \Delta r_b}{v_i} \quad (\text{F41})$$

$$\Delta r_a = \frac{1}{2}(v_b + v_i)t_a \quad (\text{F42})$$

$$\Delta r_b = \frac{1}{2}(v_i + v_e)t_b \quad (\text{F43})$$

12. A lithographic apparatus comprising:

a radiation system for providing a beam of radiation;

a substrate holder for holding a substrate having radiation sensitive material;

a projection system for projecting said beam of radiation onto a target portion of said substrate;

a setpoint generator configured to specify a trajectory to be followed by said substrate relative to said projected beam of radiation, said trajectory being characterized at least as a third order mathematical function containing information indicative of at least one of a position and an orientation as a function of time, said trajectory connecting that a first state and a second state representing boundary values for said at least position and orientation and for first and second derivatives of said at least position and orientation; and

a drive unit configured to move at least one of said substrate holder and said projection system according to the specified trajectory,

wherein said setpoint generator specifies a trajectory which provides a shortest connection in time between said first and second state, wherein said first derivative, said



second derivative, and a third derivative of said trajectory are bound by predetermined values.

13. The lithographic apparatus of Claim 12, said boundary values for the second derivatives are zero.

14. lithographic apparatus of Claim 12, further including a patterning device to configure said projected beam of radiation with a desired pattern in its cross-section.

15. The lithographic apparatus of Claim 12, wherein said setpoint generator is further configured to:

calculate separate sub-trajectories for respective dimensions of said at least position and orientation, wherein each of said sub-trajectories is characterized at least as a third order mathematical function having the shortest connection time between said first state and said second state in the respective dimension and wherein said first derivative, said second derivative, and said third derivative of each of said sub-trajectories are bound by predetermined values specified for that sub-trajectory;

select one of said sub-trajectories that has the largest connection time;

prolong the non-selected sub-trajectories to the connection time of the selected sub-trajectory while meeting the requirements of the respective predetermined values for the respective sub-trajectories; and

generate said specified trajectory by combining the selected sub-trajectory and the prolonged sub-trajectories.

16. A computer-readable medium encoded with a plurality of processor-executable instruction sequences for:

determining a position of at least one of a substrate or a projected beam of radiation; and

specifying a trajectory to be followed by said substrate relative to said projected beam of radiation, said trajectory being characterized at least as a third order mathematical

function containing information indicative of at least one of a position and an orientation as a function of time, said trajectory connecting a first state and a second state representing boundary values for said at least position and orientation and for first and second derivatives of said at least position and orientation,

wherein said specified trajectory provides the shortest connection in time between said first and second state and wherein said first derivative, said second derivative, and a third derivative of said specified trajectory are bound by predetermined values.

17. The computer-readable medium of Claim 16, wherein said boundary values for said second derivatives in the first and second state are zero.

18. The computer-readable medium of Claim 17, further including:

calculating separate sub-trajectories for respective dimensions of said at least position and orientation, wherein each of said sub-trajectories is characterized at least as a third order mathematical function having the shortest connection time between said first state and said second state in the respective dimension and wherein said first derivative, said second derivative, and said third derivative of each of said sub-trajectories are bound by predetermined values specified for that sub-trajectory;

selecting one of said sub-trajectories out of said calculated sub-trajectories based on the sub-trajectory having the largest connection time;

prolonging non-selected sub-trajectories to the connection time of the selected sub-trajectory while meeting the requirements of the respective predetermined values for the respective sub-trajectories, and

generating said specified trajectory by combining the selected sub-trajectory and the prolonged sub-trajectories.

19. The computer-readable medium of Claim 18, further including calculating at least one sub-trajectory between a position  $r_b$ , velocity  $v_b$  and zero acceleration of a first state and a second position  $r_e$ , velocity  $v_e$  and zero acceleration of the second state by:

calculating a time parameter  $t_c$  based on:

$$t_c = \frac{r_e - r_b - \Delta r_a - \Delta r_b}{v_i} \quad (G1)$$

wherein  $\Delta r_a$  and  $\Delta r_b$  are given by:

$$\Delta r_a = \frac{1}{2}(v_b + v_i)t_a \quad (G2)$$

$$\Delta r_b = \frac{1}{2}(v_i + v_e)t_b \quad (G3)$$

wherein  $v_i$  is defined by:

$$v_i = \begin{cases} -V & \text{if } r_e - r_b < 0 \\ +V & \text{if } r_e - r_b > 0 \end{cases} \quad (G4)$$

wherein  $V$  is a predetermined value defining the maximum absolute velocity, and wherein  $t_a$  and  $t_b$  are given by:

$$t_a = \begin{cases} 2\sqrt{\frac{|v_i - v_b|}{J}} & \text{if } |v_i - v_b| \leq A^2 / J \\ \frac{|v_i - v_b|}{A} + \frac{A}{J} & \text{if } |v_i - v_b| \geq A^2 / J \end{cases} \quad (G5)$$

$$t_b = \begin{cases} 2\sqrt{\frac{|v_e - v_i|}{J}} & \text{if } |v_e - v_i| \leq A^2 / J \\ \frac{|v_e - v_i|}{A} + \frac{A}{J} & \text{if } |v_e - v_i| \geq A^2 / J \end{cases} \quad (G6)$$

wherein  $A$  is a predetermined value defining the maximum absolute acceleration of the sub-trajectory and wherein  $J$  is a predetermined value defining the maximum absolute jerk of the sub-trajectory;

wherein, if  $t_c$  is greater than zero, then a sub-trajectory of a first kind comprising three parts is calculated, such that:

in the first part, a maximum velocity is built up in the shortest time  $t_a$  within the limitations of said predetermined values for the sub-trajectory,

in the second part, which has a duration of  $t_c$ , the velocity is constant and equals  $v_b$  plus the velocity built up in the first part, and

in the third part, the velocity is decreased to the velocity  $v_e$  in the shortest time  $t_b$  possible within the limitations of the predetermined values of the sub-trajectory; and

wherein, if  $t_c$  is less than zero, a sub-trajectory of a second kind comprising two parts is calculated, such that:

in the first part, a velocity is built up in the shortest time  $t_a$  possible within the limitations of the predetermined values of the sub-trajectory, and

in the second part, the velocity is decreased to the velocity  $v_e$  in the shortest time  $t_b$  possible within the limitations of the predetermined values of the sub-trajectory.

20. The computer-readable medium of Claim 19, further including performing a detection of a gap crossing move based on:

$$v_b > 0 \quad \wedge \quad v_e > 0 \quad \wedge \quad r_e > r_b \quad (G7)$$

$$v_b < 0 \quad \wedge \quad v_e < 0 \quad \wedge \quad r_e < r_b \quad (G8)$$

$$|r_e - r_b| < \Delta r(v_b) + \Delta r(v_e) \quad (G9)$$

wherein the function  $\Delta r(v)$  is given by:

$$\Delta r(v) = \begin{cases} |v| \sqrt{\frac{|v|}{J}} & \text{if } |v| \leq A^2 / J \\ \frac{1}{2} |v| \left( \frac{|v|}{A} + \frac{A}{J} \right) & \text{if } |v| \geq A^2 / J \end{cases} \quad (G10)$$

wherein a gap-crossing-move is detected if at least one of equation (G7) and equation (G8) is satisfied and equation (G9) is satisfied.

21. The computer-readable medium of Claim 20, wherein if  $t_c < 0$ , or if a gap crossing move is detected, the calculation of said sub-trajectory of the second kind comprises calculating four respective types of acceleration profiles associated with four possible sub-trajectories and said selection of the sub-trajectory out of the four possible sub-trajectories which has the shortest connection time,

wherein the first profile type is associated with the first possible sub-trajectory with a connection time T which follows from the quadratic expression:

$$\left\{ \frac{1}{2} kA \right\} T^2 + \left\{ (v_e + v_b) - \frac{A^2}{kJ} \right\} T + \left\{ -2(r_e - r_b) - \frac{(v_e - v_b)^2}{2kA} \right\} = 0 \quad (G11)$$

wherein the quadratic expression is solved for  $k=1$  and  $k=-1$ , and wherein the time duration of the first part  $t_a$  and the time duration of the second part  $t_b$  follow from:

$$t_a = \frac{1}{2} T + \frac{v_e - v_b}{2kA} \quad (G12)$$

$$t_b = \frac{1}{2} T - \frac{v_e - v_b}{2kA} \quad (G13)$$

wherein (G11) might yield more than one mathematical solution, wherein the mathematical solutions defining the first profile type satisfy:

$$t_a \geq 2A/J \quad (G14)$$

$$t_b \geq 2A/J \quad (G15)$$

$$|v_i| \leq V, \quad (G16)$$

wherein for the first profile type  $v_i$  is given by:

$$v_i = \frac{1}{2} (v_b + v_e) - \frac{A^2}{kJ} + \frac{1}{2} kAT. \quad (G17)$$

wherein the second profile type is associated with the second possible sub-trajectory with a time duration  $t_a$  of the first part which follows from the quartic expression:

$$\left\{ \frac{kJ^2}{32A} \right\} t_a^4 + \left\{ \frac{kJ}{8} \right\} t_a^3 + \left\{ \frac{v_b J}{4A} + \frac{kA}{8} \right\} t_a^2 + \left\{ v_b \right\} t_a + \left\{ \frac{(v_e + v_b)A}{2J} - \frac{v_e^2 - v_b^2}{2kA} - (r_e - r_b) \right\} = 0 \quad (G18)$$

wherein:

$$k = \begin{cases} -1 & \text{if } v_e > v_b \\ +1 & \text{if } v_e < v_b \end{cases}, \quad (G19)$$

wherein the time duration  $t_b$  of the second part and the velocity  $v_i$  at the transition from the first to the second part follow from:

$$v_i = v_b + \frac{1}{4} k J t_a^2 \quad (G20)$$

$$t_b = \frac{J}{4A} t_a^2 - \frac{v_e - v_b}{kA} + \frac{A}{J} \quad (G21)$$

wherein equation (G18) might yield more than one mathematical solution, wherein the mathematical solutions defining the second profile type satisfy:

$$0 \leq t_a \leq 2A/J \quad (G22)$$

$$t_b \geq 2A/J \quad (G23)$$

$$|v_i| \leq V \quad (G24)$$

and wherein the connection time  $T$  of the second possible sub-trajectory follows from  $T = t_a + t_b$ ;

wherein the third profile type is associated with the third sub-trajectory with a time duration  $t_b$  of the second part which follows from the quartic expression:

$$\left\{ \frac{kJ^2}{32A} \right\} t_b^4 + \left\{ \frac{kJ}{8} \right\} t_b^3 + \left\{ \frac{v_e J}{4A} + \frac{kA}{8} \right\} t_b^2 + \left\{ v_e \right\} t_b + \left\{ \frac{(v_e + v_b)A}{2J} + \frac{v_e^2 - v_b^2}{2kA} - (r_e - r_b) \right\} = 0, \quad (G25)$$

wherein:

$$k = \begin{cases} -1 & \text{if } v_e < v_b \\ +1 & \text{if } v_e > v_b \end{cases}, \quad (G26)$$

wherein the time duration  $t_a$  of the first part and the velocity  $v_i$  at the transition from the first to the second part follow from:

$$v_i = v_e + \frac{1}{4} k J t_b^2 \quad (G27)$$

$$t_a = \frac{J}{4A} t_b^2 + \frac{v_e - v_b}{kA} + \frac{A}{J} \quad (G28)$$

wherein (G25) might yield more than one mathematical solution, wherein the mathematical solutions defining the third profile type satisfy:

$$t_a \geq 2A/J \quad (G29)$$

$$0 \leq t_b \leq 2A/J \quad (G30)$$

$$|v_i| \leq V \quad (G31)$$

and wherein the connection time  $T$  of the third possible sub-trajectory follows from  $T = t_a + t_b$ ;

wherein the fourth profile type is associated with the fourth possible sub-trajectory with a time duration  $T$  which follows from the quartic expression:

$$\left\{ \frac{kJ}{16} \right\} T^4 + \left\{ v_e + v_b \right\} T^2 + \left\{ -2(r_e - r_b) \right\} T + \left\{ -\frac{(v_e - v_b)^2}{kJ} \right\} = 0 \quad (G32)$$

wherein the quartic expression is solved for  $k = 1$  and  $k = -1$ , and wherein the time duration  $t_a$  of the first part and the time duration  $t_b$  of the second part and the velocity  $v_i$  at the transition from the first to the second part follow from:

$$t_a = \frac{1}{2}T + 2\frac{v_e - v_b}{kJT} \quad (G33)$$

$$t_b = \frac{1}{2}T - 2\frac{v_e - v_b}{kJT} \quad (G34)$$

$$v_i = v_b + \frac{1}{4}kJt_a^2 \quad (G35)$$

wherein (G32) might yield more than one mathematical solution, wherein the mathematical solution defining the fourth profile type satisfies:

$$0 \leq t_a \leq 2A/J \quad (G36)$$

$$0 \leq t_b \leq 2A/J \quad (G37)$$

$$|v_i| \leq V \quad (G38)$$

22. The computer-readable medium of Claim 21, wherein if a gap-crossing-move is detected, then, if  $v_b$  and  $v_i$  have the same sign, a selection from the solutions provided by equations (G11- G17), (G18- G24), (G 25- G 31) and (G32-G38), respectively, is made such that the selection of the solutions define respectively the shortest connection time  $t_1$  and the longest connection time  $t_2$ , wherein  $t_1$  and  $t_2$  define a first connection time interval comprising possible connection times according to which the concerning sub-trajectory can be prolonged, and

wherein, if  $v_b$  and  $v_i$  have opposite signs, a selection of the solutions provided

by equations (G11-G17), (G18-G24), (G25-G31) and (G32-G38) is made such that the selection defines a connection time  $t_3$  defining a lower limit of a second connection time interval comprising possible connection times according to which the concerning sub-trajectory can be prolonged.

23. The computer-readable medium of Claim 22, wherein calculating a prolonged sub-trajectory comprises calculating a sub-trajectory with lower predetermined value or values than the corresponding predetermined value or values of the sub-trajectory to be prolonged.

24. The computer-readable medium of Claim 23, wherein if at least one of  $v_b$  and  $v_e$  equal 0, calculating a prolonged sub-trajectory comprises inserting into the sub-trajectory to be prolonged a time interval wherein the velocity is zero.

25. The computer-readable medium of Claim 24, wherein calculating a prolonged sub-trajectory comprises lowering the velocity  $v_i$  to increase  $T$ , wherein the connection time  $T$  for the sub-trajectory is defined by  $T = t_a + t_b + t_c$ , and wherein  $t_a$ ,  $t_b$  and  $t_c$  are given by:

$$t_a = \begin{cases} 2\sqrt{\frac{|v_i - v_b|}{J}} & \text{if } |v_i - v_b| \leq A^2 / J \\ \frac{|v_i - v_b|}{A} + \frac{A}{J} & \text{if } |v_i - v_b| \geq A^2 / J \end{cases} \quad (\text{G39})$$

$$t_b = \begin{cases} 2\sqrt{\frac{|v_e - v_i|}{J}} & \text{if } |v_e - v_i| \leq A^2 / J \\ \frac{|v_e - v_i|}{A} + \frac{A}{J} & \text{if } |v_e - v_i| \geq A^2 / J \end{cases} \quad (\text{G40})$$

$$t_c = \frac{r_e - r_b - \Delta r_a - \Delta r_b}{v_i} \quad (\text{G41})$$

$$\Delta r_a = \frac{1}{2}(v_b + v_i)t_a \quad (\text{G42})$$

$$\Delta r_b = \frac{1}{2}(v_i + v_e)t_b \quad (\text{G43})$$



26. A robotics system comprising:

a first movable element;

a second element;

a setpoint generator configured to specify a trajectory to be followed by said first element relative to said second element, said trajectory being characterized at least as a third order mathematical function containing information indicative of at least one of a position and an orientation as a function of time, said trajectory connecting a first state and a second state representing boundary values for said at least position and orientation and for first and second derivatives of said at least position and orientation; and

a drive unit for moving the first element with respect to the second element according to the specified trajectory,

wherein said setpoint generator specifies a trajectory which provides a shortest connection in time between said first and second state, wherein said first derivative, said second derivative, and a third derivative of said trajectory are bound by predetermined values.

27. The robotics system of Claim 26, wherein said boundary values for the second derivatives in the first and second state are zero.

28. The robotics system of Claim 26, wherein said setpoint generator is further configured to:

calculate separate sub-trajectories for respective dimensions of said at least position and orientation, wherein each of said sub-trajectories is characterized at least as a third order mathematical function having the shortest connection time between said first state and said second state in the respective dimension and wherein said first derivative, said second derivative, and said third derivative of each of said sub-trajectories are bound by predetermined values specified for that sub-trajectory;

select one of said sub-trajectories that has the largest connection time;

prolong the non-selected sub-trajectories to the connection time of the selected

sub-trajectory while meeting the requirements of the respective predetermined values for the respective sub-trajectories; and

generate said specified trajectory by combining the selected sub-trajectory and the prolonged sub-trajectories.